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Kindergartners' base-10 knowledge predicts arithmetic accuracy concurrently and longitudinally

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A B S T R A C T

Children's early knowledge of the base-10 structure of multi-digit numbers has been hypothesized to play a critical role in subsequent learning of mathematics, in particular arithmetic operations. The present study investigated the relation between base-10/place value understanding and arithmetic accuracy in early elementary school. Children were assessed in kindergarten ($N = 90$) and then a subgroup of participants was assessed again two years later in second grade ($N = 21$). Mediation analyses indicated that, in kindergarten, base-10 knowledge had a direct effect on arithmetic accuracy as well as an indirect effect through the use of a decomposition strategy. Furthermore, kindergarten base-10 knowledge had a direct effect on arithmetic accuracy in second grade and an indirect effect through second grade place-value notation understanding. Implications for understanding early mathematical development are discussed.

An understanding of the base-10 system is posited to be a critical aspect of early mathematics knowledge (Geary, 2006; Miura, 1987; NCTM, 2000; National Research Council, 2001). More specifically, it is widely believed that base-10 knowledge is necessary for accurate computation of multi-digit arithmetic problems (Fuson, 1990; Fuson & Briars, 1990; National Research Council, 2001). Errors in carrying and borrowing in written addition problems, for instance, have been attributed to a lack of understanding of base-10 and place value (Brown & Burton, 1978; Fuson, 1990; Hiebert, 1997; Ross, 1986; Varelas & Becker, 1997). Further, base-10 knowledge is related to the use of decomposition strategies, which are most efficient for solving problems with sums above 10 (Laski, Ermakova, & Vasilyeva, 2014). To date, however, discussion of the relation between base-10 understanding and arithmetic problem solving has been primarily theoretical (e.g., Fuson & Briars, 1990; Geary, Hoard, Nugent, & Bailey, 2013). In the present study, mediation analyses were used to simultaneously examine the relations among kindergartners' base-10 knowledge, addition strategies, and addition accuracy and to test whether kindergartners' early base-10 knowledge predicts accuracy on more complex multi-digit problems in second grade.

1. Base-10 and place-value notation

It takes several years for children to develop an understanding of the base-10 system and place-value notation (Carpenter, Franke, Jacobs, Fennema, & Empson, 1998; Fuson, 1986, 1988, 1992; Fuson & Briars, 1990; Ginsburg, 1989; Varelas & Becker, 1997). Before formal schooling, most children think of numbers larger than ten as collections of units rather than as groups of tens and units (Mix, Prather, Smith, & Stockton, 2014). Children's understanding of the base-10 numeric structure is typically assessed with a block-task (e.g., Miura, Okamoto, Kim, Steere, & Fayol, 1993) in which children are asked to "show" two-digit numbers using blocks that include small cubes representing single units and bars that represent ten units combined together. If children think of numbers as collections of single units, they will show a number, such as 32, using 32 individual unit cubes. If, however, children understand the base-10 structure of numbers, they are more likely to show a number, such as 32, using three ten-bars and two individual units. Between kindergarten and second grade, children increasingly use both tens- and single units to represent two-digit numbers (Miura, 1987; Miura et al., 1993; Saxton & Towse, 1998).

Thinking of multi-digit numbers as groups of tens and units should translate into later place-value notation knowledge because it lays the foundation for understanding that the numeric magnitudes represented by each digit vary based on the digit's position in a number. Suggestive of this relation, one of the most common misconceptions about place-value notation—concatenation—reflects a lack of understanding of the base-10 structure of multi-digit numbers. A child who makes a concatenation error focuses on the face-value of digits as opposed to a digit's value as a multiple of ten based on its location within a multi-digit

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number (Cobb & Wheatley, 1988; Fuson & Briars, 1990; Miura et al., 1993; Price, 1997; Ross, 1989; Varelas & Becker, 1997). For example, when asked to describe how the numeral 24 relates to 24 sticks, children making face value errors may state that the 2 in 24 represents 2 sticks and the 4 represents 4 sticks, rather than 20 and 4 (Price, 1997; Ross, 1989). Children's understanding of place-value notation and its application to arithmetic increases during elementary school. In a cross-sectional study, Varelas and Becker (1997) found that the percentage of children who traded correctly and gave the correct digit for the 10s place on a written arithmetic task increased from 56% to 77% to 98% between second and fourth grades. In the present study, the relation between early base-10 knowledge and later place-value notation understanding was examined empirically using a longitudinal design.

2. Base-10, addition strategies, and accuracy

To be successful on more complex problem solving in math, children must first learn to accurately and efficiently solve simple arithmetic problems in early elementary school (Cowan et al., 2011; Jordan, Kaplan, Olah, & Locuniak, 2006). Children can arrive at solutions to addition problems through various types of strategies. When asked to solve problems without paper and pencil, children typically use one of three types of addition strategies: counting, decomposition, and retrieval (Geary, Bow-Thomas, Liu, & Siegler, 1996a, 1996b; Geary, Fan, & Bow-Thomas, 1992; Shrager & Siegler, 1998). Counting strategies involve enumerating both of the addends or counting-up from one of the addends. The retrieval strategy involves recalling the solution to a problem as a number-fact stored in memory, rather than active computation. Decomposition involves transforming the original problem into two or more simpler problems, and often begins with solving for ten first (e.g., base-10 decomposition: solving $6 + 5$ by adding 6 and 4 to get to 10, and then 1 more).

A decomposition strategy is useful for solving arithmetic problems, particularly when the problems involve sums above ten and/or double-digit addends (Ashcraft & Stazyk, 1981; Torbeyns, Verschaffel, & Ghesquiere, 2004). Children and adults who frequently use decomposition to solve arithmetic problems tend to have higher math performance and overall math achievement scores than those who depend on counting strategies (Carr & Alexeev, 2011; Carr, Steiner, Kyser, & Biddlecomb, 2008; Geary, Hoard, Byrd-Craven, & DeSoto, 2004; Fennema, Carpenter, Jacobs, Franke, & Levi, 1998). A recent study found that the frequency with which first graders' use a decomposition strategy predicted their accuracy on complex addition problems and mediated cross-national differences in accuracy on these complex arithmetic problems (Vasilyeva, Laski, & Shen, 2015).

Kindergartners who use decomposition tend to have a better understanding of the base-10 structure of the number system than those who do not (Laski et al., 2014). This relation makes sense theoretically. Consider, for example, addition problems with single-digit addends that require carryover into the tens place (e.g., $7 + 5$). Having a base-10 understanding of two-digit numbers (e.g., $12 = 10 + 2$) may facilitate the use of decomposition in solving this problem: $7 + 5 = (7 + 3) + 2 = 12$. Similarly, better understanding of base-10 structure may facilitate use of decomposition strategy in solving problems with multi-digit addends, with or without carryover (e.g., $23 + 14$). In order to use a decomposition strategy that involves adding tens, then ones, then combining the results, a child must know which digits represent the tens and be able to increment by tens rather than ones. Therefore, it is not surprising that understanding of base-10 structure predicts the use of decomposition. Identifying base-10 knowledge as a predictor of decomposition strategy use by children is noteworthy because this strategy has been shown to lead to higher arithmetic accuracy (e.g., Geary et al., 2004). Importantly, however, the relation among base-10 knowledge, use of decomposition strategy, and arithmetic accuracy has not been investigated in the context of a single study within the same group of children. Thus, no direct evidence for the theoretical relation between these three aspects of mathematics knowledge exists. The present study empirically tested this relation.

3. The present study

The present study had two primary goals. The first goal was to simultaneously examine the relations among kindergartners' base-10 knowledge, addition strategies, and addition accuracy. Based on the analysis above, we expected that the extent to which children represented double-digit numbers as tens and ones, rather than as a collection of units, would predict their accuracy on addition problems and that this relation would be mediated by the frequency with which they used a decomposition strategy.

The second goal of the study was to examine longitudinally the relation between early base-10 knowledge and later mathematics performance. A subgroup of kindergartners was followed to second grade and asked to answer place value notation questions as well a set of more complex arithmetic problems. We expected that kindergartners' base-10 knowledge would predict their understanding of place-value notation two years later. Further, we expected that early base-10 knowledge would influence later arithmetic accuracy through place-value notation understanding because place-value notation understanding is essential for solving complex arithmetic problems that involve multi-digit addends and carrying and borrowing.

4. Method

4.1. Participants

The present study included a group of kindergartners ($N = 90$; *Mean age* = 6;1 years, $SD = 0.34$). In addition to testing the full sample in kindergarten, we were able to test a subset of the sample two years later when children were in second grade ($N = 21$; *Mean age* = 8;3 years, $SD = 0.42$). Analyses indicated no differences on kindergarten

Table 1
Coding scheme for addition strategies.

Category	Definition	Behavioral/verbal cues	Example
Counting	Enumerating each unit in one or both of the addends	Child verbally counted by one or exhibited counting through behavioral cues during problem solving or reported enumerating addend(s) when describing the solution.	6 + 5… "7, 8, 9, 10, 11" "I counted 5 from 6."
Decomposition	Transforming the original problem into simpler problems, using base-10 properties or previously memorized number facts	Child reported several steps involving breaking the original addends into smaller numbers. This could be observed during problem solving or during the child's explanation.	6 + 5… "6 + 4 = 10, 10 + 1 = 11"
Retrieval[a]	Recalling a required number fact from memory	Child reported the answer within 3 s with no overt evidence of counting or decomposition stated that he/she "just knows the answer"	6 + 5… 11, "I just know it."
Other	None of the above	Child reported guessing or not knowing, or reported a strategy that was unclear and could not be clarified by further prompting	"I don't know."

[a] Retrieval is only used on single-digit problems.

Table 2
Percent correct at Time 1 and Time 2.

	Time 1			Time 2	
	Single-digit	Multi-digit	Overall	Place value	Arithmetic
Full sample $n = 90$	69% (28%)	55% (35%)	63% (29%)		
T2 sample $n = 21$	69% (30%)	57% (34%)	64% (30%)	65% (30%)	63% (22%)

measures of math knowledge between those children followed to second grade and others from the full kindergarten sample, all *p*-values > 0.05. Children were recruited from private and public schools serving middle- to higher-income families.

4.2. Measures of Base-10 and Place Value.

4.2.1. Time 1

In the base-10 block-task (Miura, 1987), an experimenter presented kindergartners with unit-blocks and 10-blocks and explained that the blocks could be used to show numbers. After two practice trials, each child was given five test trials. On each test trial, the experimenter presented a child with a different number card and asked the child to show the number using blocks. For each trial, the experimenter coded whether the child used a canonical base-10 presentation, which involved using the largest possible number of 10-blocks to represent 10s and unit-blocks to represent ones (e.g. showing 23 with two 10-blocks and three unit-blocks), or used other strategies (e.g., only a collection of units) to represent the given number.

4.2.2. Time 2

Second graders completed five place-value notation problems. Three problems required children to determine the largest or smallest number among a group of numbers presented either as Arabic numerals (e.g., 10,101) or as groups of tens and ones (e.g., 3 tens and 28 ones). One problem required children to identify the tens place in multi-digit numbers. An additional problem required children to generate the largest possible number using a set of given digits. Children's mean accuracy on the five problems was calculated.

4.3. Measures of arithmetic accuracy

4.3.1. Time 1

Kindergartners were presented with 24 addition problems, one at a time. Half of them were single-digit problems ($5 + 6 =$) and the other half were multi-digit problems (e.g., $15 + 8 =$). The experimenter read each problem aloud and then gave children as much time as needed to solve the problem. Children were not provided with any supplies, such as paper or pencil, but were permitted to use their fingers or to count aloud. Children's accuracy for each problem was coded as 1 or 0 and averaged across all 24 items.

4.3.2. Time 2

Second graders completed 10 double- and mixed-digit addition and subtraction problems. Half of these were *contextualized* within a story (e.g., “A grocery store had 89 bananas. They sold 27 bananas on Monday and 34 bananas on Tuesday. How many bananas were left in the grocery store on Wednesday?”). The other half were *decontextualized*; children were presented with double-digit addition, subtraction, and missing term problems (e.g., 42–29 =) using only numerical symbols. All problems were presented in one of two randomized orders. Children could solve the problems either mentally or with the paper and pencil provided. Children's accuracy was calculated across these two problem types.

4.4. Measure of strategies

Kindergartners' approach to solving the problems was coded using experimenters' notes of overt behavior and children's retrospective reports of strategy use. Strategies were coded as counting, decomposition, retrieval, or “other.” A more complete description of the strategies can be found in Table 1. The “retrieval” code was used only on single-digit problems because it has been generally accepted that retrieval only applies to stored number facts involving single-digit numbers (e.g., Geary et al., 2004). In situations when children used more than one strategy in executing decomposition they were coded as only using decomposition. For example, the decomposition code was used if a child retrieved the answer to a simpler problem and then used count on to reach his final answer (e.g., to solve $5 + 7$, a child might recall that $5 + 5 = 10$ and then count on 11, 12). Raters discussed all situations in which children's reported strategy conflicted with their observed behavior and the raters together agreed on the final strategy code.

5. Results

While the primary analyses involved examining the relation between base-10 knowledge, strategy use, place value and arithmetic accuracy, we began by examining descriptive statistics on all the measures in kindergarten and second grade. Table 2 presents kindergarteners and second-graders' accuracy on the tasks administered at Time 1 and Time 2. In terms of base-10 knowledge in kindergarten, children used a canonical base-10 representation on 48% ($SD = 42\%$) of trials. The subgroup of children who were followed to second grade answered 65% ($SD = 30\%$) of the place-value notation problems correctly. In terms of arithmetic accuracy, children accurately solved more than half of the arithmetic problems correctly at both grade levels: 63% ($SD = 29\%$) in kindergarten and 63% ($SD = 22\%$) in second grade. In terms of addition strategy choice in kindergarten, children's predominant strategy was counting, which was used on 65% ($SD = 31\%$) of problems. Kindergartners did, however, use retrieval and decomposition to solve some problems: 15% ($SD = 19\%$) and 10% ($SD = 18\%$) of problems, respectively. Table 3 presents the percentage of problems of which children used each type of strategy at Time 1. Thus, in general, children demonstrated moderate performance on all measures – they were neither at floor nor ceiling on any measure – and there was sufficient variability across children, allowing us to test for predictors of individual differences in strategy choice and arithmetic accuracy.

Table 3
Percent of trials on which strategies were used.

	Task 1	Task 2		
	Base-10 canonical representation	Retrieval	Decomposition	Counting
Full sample $n = 90$	48% (42%)	15% (19%)	10% (18%)	65% (31%)
T2 sample $n = 21$	63% (44%)	14% (15%)	8% (17%)	70% (30%)

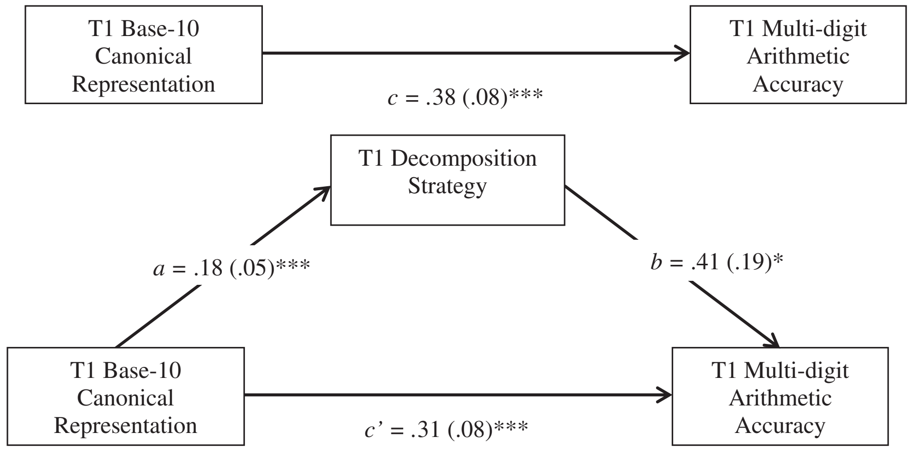

Fig. 1. Concurrent relations: direct and indirect effects of base-10 knowledge on arithmetic accuracy among kindergartners. *Note*. *$p < 0.05$, **$p < 0.01$, ***$p < 0.001$.

Standard deviations are presented in parentheses.

To examine the relations among base-10 knowledge, use of decomposition, and arithmetic accuracy in kindergarten, we conducted a bias-corrected bootstrapping mediation analysis (Dearing & Hamilton, 2006) using the SPSS macro "PROCESS" (Preacher & Hayes, 2008). This analysis allowed us to simultaneously examine direct effects as well as whether the influence of base-10 knowledge on multi-digit arithmetic accuracy occurred through the use of decomposition. Because it was possible that children's performance on all three measures was indicative of their general math ability, we added a covariate—a measure of children's fluency on simple addition problems—to reduce the possibility of this confound. Fluency was calculated as the percentage of single-digit problems on which retrieval was used correctly; the mean fluency score at Time 1 was 20% ($SD = 24\%$).

As shown in Fig. 1, base-10 canonical representation was related to both multi-digit arithmetic accuracy, $c = 0.38$, $s_c = 0.08$, $p < 0.001$, and use of a decomposition strategy, $a = 0.18$, $s_a = 0.05$, $p < 0.001$. The association between base-10 canonical representation and multi-digit arithmetic accuracy was weaker, however, when both base-10 canonical representation and use of decomposition were included in a single model predicting multi-digit arithmetic accuracy: base-10 canonical representation, $c' = 0.31$, $s_b = 0.08$, $p < 0.001$, and decomposition, $b = 0.41$, $s_b = 0.19$, $p = 0.04$ The product of the coefficients (ab) for the indirect path from base-10 knowledge to arithmetic accuracy by way of frequency of using a decomposition strategy was significant (point estimate = 0.07, 95% bias-corrected confidence interval = [0.02 to 0.15]).

Next we examined whether individual differences in base-10 knowledge in kindergarten had a long-term effect on children's arithmetic accuracy, and whether there was an indirect effect through place-value notation understanding. We ran another mediation analysis using the bootstrapping method with bias-corrected confidence estimates. Again, to reduce the possibility that general math ability explained any associations, we used kindergarten fluency as a covariate on the outcome (second grade arithmetic accuracy); the Time 1 mean fluency for the restricted sample was 18% ($SD = 19$).

As shown in Fig. 2, kindergarten base-10 canonical representation was associated with both second-grade arithmetic accuracy, $c = 0.35$, $s_c = 0.14$, $p = 0.02$, and second graders' place-value notation accuracy, $a = 0.22$, $s_a = 0.10$, $p = 0.04$. When kindergarten base-10 canonical representation and second-grade place value accuracy were include in a single model, second-grade arithmetic accuracy was associated with second-grade place value, $b = 0.92$, $s_b = 0.23$, $p = 0.001$, but the relation between kindergarten base-10 canonical representation and second-grade addition accuracy, became insignificant, $c' = 0.18$, $s_b = 0.11$, $p = 0.14$. The product of the coefficients (*ab*) for the indirect

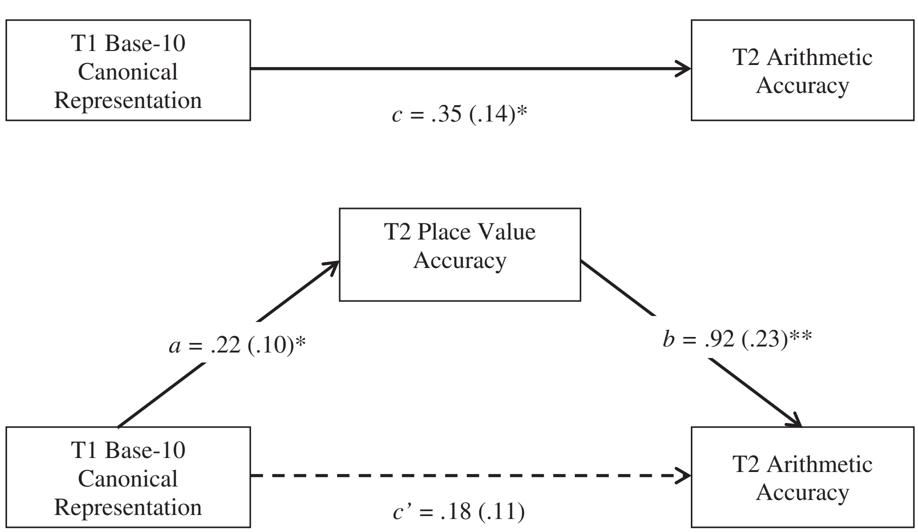

Fig. 2. Longitudinal relations: direct and indirect effects of base-10 knowledge on arithmetic accuracy from kindergarten to second-grade. *Note*. *$p < 0.05$, **$p < 0.01$, ***$p < 0.001$.

path from kindergarten base-10 knowledge to second grade arithmetic accuracy through second grade place-value notation accuracy was significant (point estimate = 0.20, 95% bias-corrected confidence interval = [0.04 to 0.52]).

6. Discussion

The present study provides a bridge between two bodies of literature about early mathematics learning. One set of studies has documented the relation between the use of decomposition and arithmetic accuracy (Carr & Alexeev, 2011; Geary et al., 2004; Fennema et al., 1998; Vasilyeva et al., 2015). Other work has established the relation between children's understanding of the base-10 structure and use of decomposition strategies (Laski et al., 2014). These separate lines of research suggested that the use of decomposition might mediate the relation between base-10 understanding and arithmetic accuracy. That is, children who have a better understanding of base-10 structure will be more likely to use decomposition in solving arithmetic problems, which, in turn, will be associated with higher accuracy on these problems.

The results of the present study provide support for the theoretical relation among these three aspects of math knowledge. It is noteworthy that the findings were consistent with the theoretical predictions, suggesting they were not spurious or due to chance. First, they demonstrate that individual differences in base-10 knowledge at least partially explain individual differences in arithmetic accuracy both concurrently and longitudinally. Further, the mediation analyses provide insight into the mechanisms by which children's understandings of numeric structure influences their performance on computation tasks. In kindergarten, base-10 knowledge supports execution of a decomposition strategy, which, in turn, leads to greater addition accuracy on problems with sums above ten. Over time, initial base-10 knowledge leads to a better understanding of place-value notation, which is related to greater accuracy on arithmetic problems involving multi-digit addends. Identifying these paths has implications for understanding the development of mathematics skill.

Generally, the findings serve as another example of the relation between conceptual and procedural knowledge in mathematics. Previous research has shown conceptual knowledge can influence procedural knowledge or vice versa (for a review, see Rittle-Johnson & Siegler, 1998). To better understand this iterative relation between concepts and procedures it is important to map out the sequence with which skills emerge and how they influence each other. In this case, base-10 knowledge was found to be a precursor of a decomposition strategy and an abstract understanding of place-value notation, both of which predicted arithmetic accuracy. This suggests that efforts focused on improving children's base-10 knowledge in early childhood could reduce individual differences in other mathematics outcomes.

Importantly, research indicates that children's knowledge of base-10 is quite malleable. The age at which children accurately use ten-blocks and unit cubes to represent two-digit numerals seems to depend, in part, on their instructional experiences (Fuson & Briars, 1990; Fuson, Smith, & Lo Cicero, 1997; Hiebert & Wearne, 1992; Miura et al., 1993; Varelas & Becker, 1997). For example, Saxton and Cakir (2006) found that providing children with practice using count-on or partitioning sets (e.g., 12 into 5 and 7 or 9 and 3) increased children's knowledge of base-10. Further, cross-national studies indicate no differences in East Asian and American children's use of base-10 representations upon school entry (Vasilyeva, Laski, Ermakova, Lai, Jeong, & Hachigan, 2015), whereas these differences have been documented later, after children received math instruction (e.g., Miura et al., 1993). Recent work suggests improvement in the base-10 knowledge of American first graders in the last 20 years, corresponding with the increased instructional emphasis on children's understanding of base-10 structure (Vasilyeva, Laski, & Ermakova, 2015). The present study suggests that a focus on base-10 knowledge in early childhood is well-founded and could have lasting effects on children's arithmetic performance.

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